# Engineering Notes

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## Modifying a General Aviation Airfoil for Supercritical Flight

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### Introduction

IT is well known that for a given operating lift coefficient there will be one freestream Mach number, the critical Mach number, at which sonic flow will first be achieved at some point on an airfoil. With increasing freestream Mach numbers, this point will expand continuously into a supersonic "bubble," normally terminated by a shock wave, over the upper surface of the section.

Today's modern high-speed subsonic transport aircraft all operate well into this supercritical transonic regime. It is not uncommon for a large part of the upper surface of their airfoils to be wetted with supersonic flow; these supersonic "bubbles" are almost always terminated by a shock wave (the Mach number just ahead of the shock is typically from 1.2 to 1.3). This seemingly inevitable phenomenon is clearly depicted in the pressure distribution shown in Fig. 1, which illustrates the computed flow about a low-speed high-lift general aviation [GA(W)-2] airfoil at $M_\infty = 0.77$, $C_L = 0.4$, with $Re = 6 \times 10^6$.

The presence of this shock is characteristic of the transonic flow regime, and its effect on the drag characteristics of a given airfoil is usually dramatic. The shock foot/boundary-layer interaction is internally quite complex; and the pressure rise associated with the shock wave often leads to the macroscopic result of boundary-layer separation if the shock is of more than moderate strength.

An important characteristic of an airfoil is its drag-rise Mach number, that is, the Mach number where the airfoil drag becomes unacceptably large. The goal of this study is to show that one may delay this drag-rise threshold. Thus higher flight speeds may be achieved, improving fuel efficiency—a factor of paramount concern today.

These considerations, basic to modern aircraft design, gave great impetus in the mid-1950s to seeking solutions to the transonic flow equations that would yield shock-free flow. While these shock-free configurations were at first only assumed to exist, they were proven to be mathematically isolated from one another by Morawetz.[1] Thus any deviation from a designed shock-free configuration must have shocks in the flow, although possibly very weak ones. In a pioneering study in 1978 using the "fictitious gas" method of Sobieczky,[2] Sobieczky et al.[3] found that a great number of shock-free configurations could be found in two dimensions, with implications that the same would be true in three dimensions as well.

The purpose of this Note is to illustrate the practicality and efficiency of the above design method in obtaining a useful airfoil design. While we deal here with strictly two-dimensional considerations, the extension to three-dimensional wing design is indicated.

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### Fictitious Gas Concept and Design Method

The design method used in this work is the elegant, nearly direct design method of Sobieczky.[2] In two dimensions, it provides a highly effective computational procedure for obtaining an airfoil shape that will yield shock-free performance, given a suitable starting configuration. As was previously stated, an airfoil operating in the transonic regime will be topped by a so-called sonic bubble. Inside this sonic surface the governing equations for supersonic flow display hyperbolic behavior, and on the sonic surface where $M = 1$ the equations are parabolic. Outside this region and over the remaining airfoil surface, the flow is subsonic, governed by an elliptic equation. The results obtained by solving elliptic equations for the flow parameters are inherently smooth and continuous, unlike those for the hyperbolic equations of the sonic bubble. The procedure used here introduces into the computations a fictitious gas law that will revert the hyperbolic regions to elliptic behavior. This is achieved by merely defining a new relationship between density and the local speed for supersonic Mach numbers.[2,3] Thus we computationally place the airfoil in a fictitious gas flow to obtain a solution that by its very nature provides smooth data on the sonic line. It should be noted that the local flow speed and flow angles so calculated for the sonic line are correct, since the correct equations are used until the type change at $M = 1$, and the rest of this data inside the sonic region is discarded. A simple example of the fictitious gas method is to use the density at its correct value for subsonic flow, and held constant at its sonic value for supersonic flow. The final step is a simple marching procedure that begins with the velocity potential and flow angle found using the fictitious gas on the sonic line. These may or may not be consistent with shock-free flow. Using the method of characteristics, we calculate inward from the sonic line through the supersonic region, seeking the streamline that continues the original airfoil surface into the supersonic region. If this marching is successful, a shock-free airfoil has been found. If it is not, the sonic line data are not compatible with shock-free flow (i.e., a limit line intervenes).

This design procedure, while just described for inviscid potential flow, has been shown by Nebeck and Seebass[4] to work equally well in the presence of viscous effects. The Grumfoil analysis algorithm[5] couples the inviscid potential solution of Jameson[6] with the interactive boundary-layer calculation of Green et al.[7] Thus, in addition to the potential solution, a highly reliable boundary-layer displacement thickness is calculated, and wake curvature effects are correctly accounted for. When these computations are interfaced with the fictitious gas redesign code, as incorporated by Nebeck and Seebass,[4] the resulting shock-free design is of practical interest, since it was found in the presence of the actual boundary layer.

### Airfoil Design

Having described the global design procedure and algorithm, we now illustrate the actual steps taken to modify the GA(W)-2 in this study. The GA(W)-2 was chosen as a parent airfoil because of its published high $C_{L_{max}}$ (approximately 2.0 at $Re=6\times10^6$) and its projected supercritical performance.[8] A shock-free design was sought for $M_\infty=0.77$ and $C_L\simeq0.4$, at a flight Reynolds number of $6\times10^6$, typical cruise conditions for an executive jet. With these conditions in mind, an initial analysis of the GA(W)-2, which is 13% thick, was performed. The resulting pressure distribution is shown in Fig. 1. As can be clearly seen, the supersonic region is terminated by a strong shock. The upper surface skin friction vanishes at $x/c=0.66$, indicating that the flow is separated.

Since the GA(W)-2 is quite thick (13%) and a thinner airfoil seemed acceptable, it was decided to alter its upper surface shape. Both its thickness and upper surface curvature were reduced slightly with a simple algebraic function. The forward 10% of chord was unchanged, as it was felt that this would preserve the low-speed high-lift performance.

With this new baseline airfoil, designated the GA(W)-2MOD, shock-free design attempts were immediately successful. Lift coefficients were increased from the expected 0.4 to 0.45 and finally to 0.50 at $M_\infty=0.77$. At higher-lift coefficients, and at Mach numbers greater than 0.7725, no shock-free solution could be found. Thus we took the final design to be at $M_\infty=0.7725$ with $C_L=0.50$; this we called the GA(C)-250, which is 11.7% thick.

We should note that there is a strong interplay between the baseline airfoil and the fictitious gas law used. These are at the discretion of the designer, provided they result in airfoils that satisfy constraints on thickness, shape, pitching moment, etc.

## Results and Discussion

The final step of our procedure is the analysis of the designed airfoil, first at its design point, and then at several off-design Mach numbers. At its design point, the GA(C)-250 clearly exhibits shock-free behavior, as seen in Fig. 2, with its real gas pressures closely resembling the elliptic fictitious gas pressure. Most importantly, however, we notice that the drag of this 11.7%-thick airfoil is 106 counts, while that for the original 13%-thick GA(W)-2 at $M_\infty=0.77$, but at a lower $C_L$ of 0.4, was 216 counts.

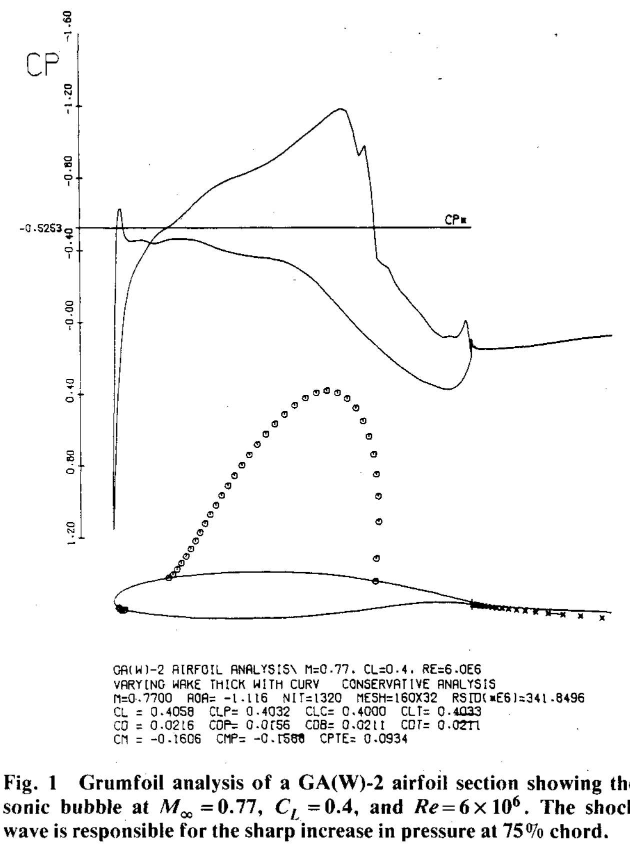


**Fig. 1 Grumfoil analysis of a GA(W)-2 airfoil section showing the sonic bubble at $M_\infty=0.77$, $C_L=0.4$, and $Re=6\times10^6$. The shock wave is responsible for the sharp increase in pressure at 75% chord.**

It seemed more meaningful, however, to compare our shock-free airfoil with another of nearly the same thickness. In addition, it was hoped that the strength of the shock on the comparison airfoil would be more moderate, causing less severe boundary-layer separation, and hence the computations would be more reliable. To this end, the original GA(W)-2 was thinned to match the GA(C)-250 thickness of 11.7% by simply reducing its ordinates uniformly, with its camber line and thickness distribution preserved. A comparison of the three airfoils is shown in Fig. 3.

An analysis of this thinned GA(W)-2, which we called the GA(W)-2T, was run at $M_\infty=0.7725$ and $C_L=0.50$. We note that the drag count for this airfoil is 201. An overlay of the airfoil shapes and pressure distributions for the GA(C)-250 and GA(W)-2T is shown in Fig. 4. This striking comparison shows clearly the small differences between the two sections, and their great effect upon the flow at these high speeds.

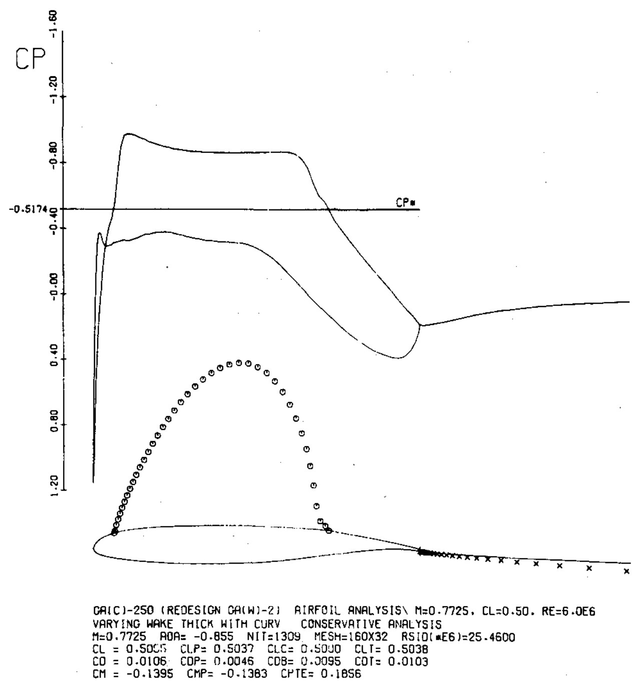


**Fig. 2 Grumfoil analysis shows the shock-free performance of the GA(C)-250. The final design thickness here is 11.7%.**

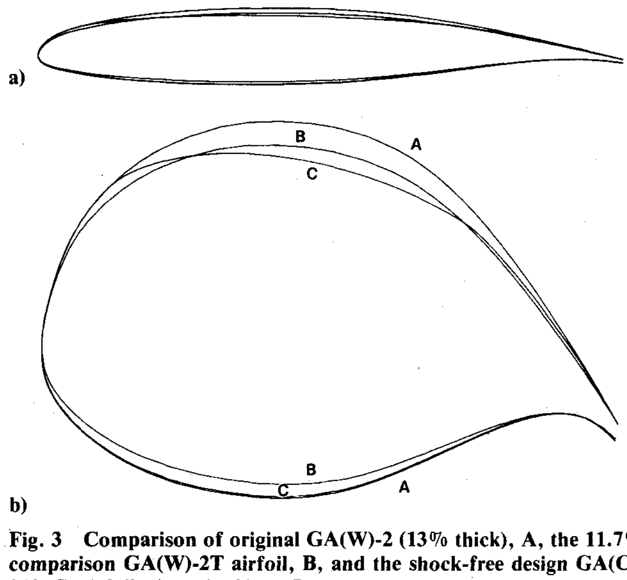


**Fig. 3 Comparison of original GA(W)-2 (13% thick), A, the 11.7% comparison GA(W)-2T airfoil, B, and the shock-free design GA(C)-250, C, airfoil: a) $y=1x$; b) $y=5x$.**

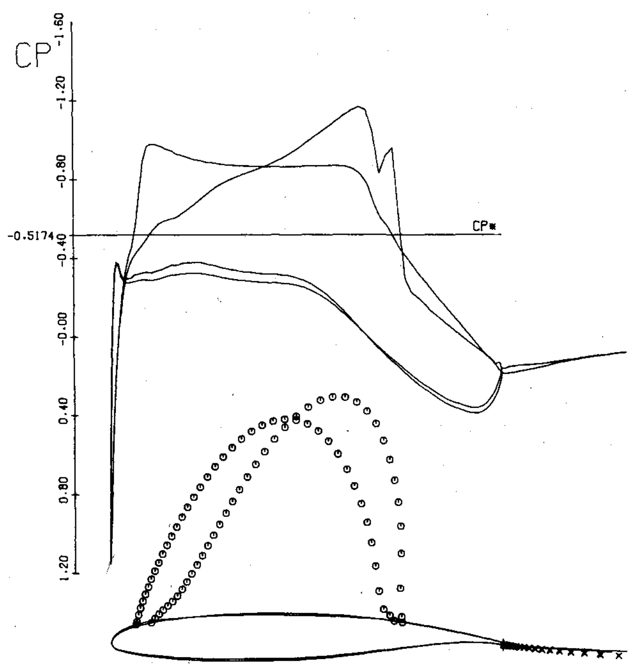


**Fig. 4 Overlay of the GA(C)-250 (shock-free) and GA(W)-2T airfoil shapes and pressure distributions. Conditions here are $M_\infty = 0.7725$, $C_L = 0.50$, and $Re = 6 \times 10^6$.**

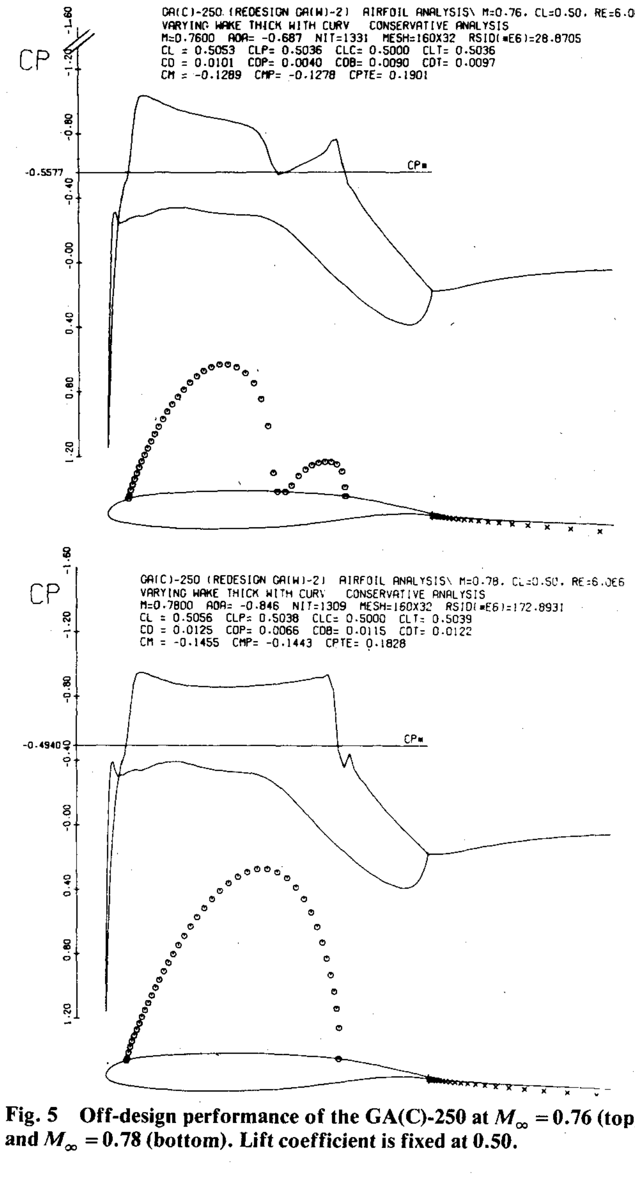


**Fig. 5 Off-design performance of the GA(C)-250 at $M_\infty = 0.76$ (top) and $M_\infty = 0.78$ (bottom). Lift coefficient is fixed at 0.50.**

Figure 5 reveals the performance of the GA(C)-250 at off-design Mach numbers. The lift coefficient here is fixed at 0.50. At higher Mach numbers, while a shock begins to form, it is of moderate strength up to $M_\infty = 0.78$. Below the design Mach number, the sonic region splits into two separate bubbles; here again the shocks are relatively weak, with no resulting boundary-layer separation.

While two-dimensional in character, the results of this work have a natural extension to three-dimensional wing design. Here, it is not the redesigned airfoil but its parent baseline that is of interest. Since this baseline has yielded a shock-free design at the Mach number and lift coefficient of interest, it represents a logical starting point for a baseline wing section for input into three-dimensional design codes. A detailed description of this process can be found in Refs. 9 and 10. The baseline produced in this study is utilized in the author's present wing design efforts, as described in Ref. 11.

## Conclusion

The redesign of a low-speed high-lift general aviation airfoil [GA(W)-2] to be shock-free at transonic flight regimes has been described. With its forward 10% of chord as well as its entire lower surface left unchanged, the section's low-speed performance should not be adversely affected. Employing the fictitious gas shock-free design method, the computations required only 350 s of CDC 7600 CPU time. The redesigned airfoil is shown to have a drag count of less than half that of the equally thick section obtained by proportionally reducing the GA(W)-2 ordinates.

## Acknowledgments

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